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Fourth Semester B.Sc. Degree Examination, July 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, FOURIER SERIES AND FOURIER TRANSFORMS

(2014–2017 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION I

All the first 10 questions are compulsory. They carry **1** mark each.

1. Find the principal value of i^i.
2. State whether $f(z) = \overline{z}$ is analytic or not.
3. Find the isolated singular points of the function $f(z) = \dfrac{z+1}{z^3(z^2+1)}$.
4. Show that $\int_C \dfrac{z^3}{z-3} dz = 0$ where C is the circle $|z| = 2$.
5. State Liouville's theorem.
6. Find the residue of $f(z) = \dfrac{z^2-1}{z^2+z}$ at $z = 0$.

P.T.O.

7. Express $\frac{1-i}{1+i}$ in polar form.

8. Write down the Euler formulae for calculating the Fourier coefficients of function $f(x)$ of period 2π.

9. What is the standard form of Fourier series for an even function?

10. State the sufficient conditions for the existence of Fourier transform.

(10 × 1 = 10 Marks)

SECTION II

Answer any **eight** questions. Each question carries **2** marks.

11. Show that $u(x, y) = y^3 - 3x^2y$ is harmonic.

12. Show that the complex function $f(z) = 2x^2 + y + i(y^2 - x)$ is not analytic at any point.

13. If a function f is analytic at a point, show that it is continuous there

14. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along the path $y = x$.

15. Verify Cauchy's theorem for the function $f(z) = z$ and C is the circle $|z| = 1$.

16. Find the singular points of the function $\frac{z+1}{z^3(z^2+1)}$.

17. Expand $f(z) = \frac{z-1}{z^2}$ as a Taylor series about $z = 1$.

18. Find the residue of $f(z) = \frac{z^2}{(z-1)^3}$ at its poles.

19. Using Cauchy's residue theorem, evaluate $\int_C \frac{e^z}{\sin z} dz$ where C is the circle $|z| = 1$.

20. Find the Fourier series of $f(x) = x$ for $0 < x < 2\pi$.

21. Find the Fourier sine series for the function $f(x) = \pi - x$ in $0 < x < \pi$.

22. Find the Fourier transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.

(8 × 2 = 16 Marks)

SECTION III

Answer any **six** questions. Each question carries **4** marks.

23. If a function is analytic, show that it is independent of $\bar{z}$.

24. Find the analytic function $f(z) = u + iv$ whose real part is $e^x(x\cos y - y\sin y)$.

25. Find the singularities of the function $f(z) = \frac{z^2 - z}{(z+1)^2(z^2+4)}$ and hence find the corresponding residues.

26. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2n)!}{(n!)^2}(z - 3i)^n$.

27. Find the value of the integral $\int_C \frac{3z^2 + 2}{(z-1)(z^2+9)} dz$ taken counter clockwise around the circle $|z| = 4$.

28. Evaluate the improper integral $\int_0^{\infty} \frac{dx}{1+x^2}$ using residues.

29. Find a Fourier series that represents $f(x) = |x|$ in $[-\pi, \pi]$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}$.

30. Expand the function defined by $f(x)=\begin{cases}0 \text{ for } -2<x<0\\ x \text{ for } 0\le x<2\end{cases}$ as a Fourier series on $[-2,2]$.

31. Find the Fourier transform of $f(x)=e^{\frac{x^2}{2}}$. What is your inference?

(6 × 4 = 24 Marks)

SECTION IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Let f denote the function defined by $f(z)=\begin{cases}\frac{(\bar{z})^2}{z} \text{ when } z\neq 0\\ 0 \text{ when } z=0\end{cases}$

Show that f is not differential at $z=0$ even though Cauchy–Riemann equations are satisfied there.

(b) If $u-v=(x-y)(x^2+4xy+y^2)$, find an analytic function $f(z)=u+iv$ in terms of z.

33. (a) Find the Laurent's series expansion for the function $f(z)=\frac{1}{z(z^2-3z+2)}$ valid for $1<|z|<2$.

(b) Evaluate $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$.

34. Find a Fourier series to represent $f(x)=x-x^2$ in $(-\pi,\pi)$ and hence show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$.

35. (a) Show that the Fourier transform is a linear operator.

(b) Find the Fourier transform of f (x), where $f(x)=\begin{cases}1 \text{ if } |x|<1\\ 0 \text{ otherwise}\end{cases}$.

(2 × 15 = 30 Marks)